transformation, because the original Eq. (1) can be used much more easily, provided that the correct values are used. By this consideration, it is unnecessary to indicate the stationary and moving frame as done by Adarkar. In conclusion, to my knowledge the problem in my previous note is mathematically well posed.

**Reference**

[1] Chen, N. H., "Simplified solutions for ablation in a finite slab," AIAA J. 3, 1148–1149 (1965).

# Comment on "Validity of Integral Methods in MHD Boundary-Layer Analysis"

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HEYWOOD and Moffatt[1] have given a detailed and lucid discussion on the difficulties involved in applying the so-called integral methods to MHD problems. Here, we wish to supplement their work by emphasizing the fact that the difficulties are inherent in nature, and much more deeply rooted than usually suspected. We wish to exemplify this by the following observation.

Consider the MHD Rayleigh problem[2] governed by the following system:

$$\begin{cases} \partial u/\partial t = \nu(\partial^2 u/\partial y^2) + a^2(\partial b/\partial y) \\ \partial b/\partial t = \eta(\partial^2 b/\partial y^2) + (\partial u/\partial y) \\ t = 0: \quad b = u = 0 \\ y = 0: \quad u = 1, \quad b = 0 \quad \text{(perfectly insulating)} \\ y \to \infty: \quad u \to 0, \quad b \to 0 \end{cases}$$

It is not necessary to explain the meaning of the symbols used here, except to indicate that $a^2(\partial b/\partial y)$ and $\partial u/\partial y$ represent the interactions between the magnetic and the flow fields. Now if we integrate this system with respect to $y$ from 0 to $\infty$, the resulting integral relations are

$$\frac{d}{dt}\int_0^\infty u\,dy = -\nu\left.\frac{\partial u}{\partial y}\right|_{y=0}$$

$$\frac{d}{dt}\int_0^\infty b\,dy = -\eta\left.\frac{\partial b}{\partial y}\right|_{y=0} - 1$$

The influence of the magnetic field on the flow field is lost completely, whereas that of the flow field on the magnetic field is over-simplified to a constant term 1. It is obvious that regardless of which trial curves one uses for $u$ and $b$, the result will not be acceptable.†

It is interesting, however, to notice that the same problem, if the induced magnetic field is neglected because of small viscous diffusivity compared to the magnetic diffusivity, is governed by one single equation

$$\partial u/\partial t = \nu(\partial^2 u/\partial y^2) - cu$$

which yields the integral relation

$$\frac{d}{dt}\int_0^\infty u\,dy = -\nu\left.\frac{\partial u}{\partial y}\right|_{y=0} - c\int_0^\infty u\,dy$$

In this equation, the interaction is decently represented. As a matter of fact, a trial solution of the form

$$u = \operatorname{erfc}\left[\frac{y}{2(\nu)^{1/2}\delta(t)}\right]$$

yields results that check very well with the exact solution for all $y$ values at small times. For large time, the check is very good near the wall, as expected.

**References**

[1] Heywood, J. B. and Moffat, W. C., "Validity of integral methods in MHD boundary-layer analyses," AIAA J. 3, 1565–1567 (1965).

[2] Bryson, A. E. and Rościszewski, J., "Influence of viscosity, fluid conductivity and wall conductivity in the magnetohydrodynamic Rayleigh problems," Phys. Fluids 5, 175–183 (1962).

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† The two integral relations are correct, of course. It is only incorrect (not even as an approximation) here to try to obtain local behavior from them. In other words, it is true that the net effect of the interaction on the fluid as a whole is represented by the two constants 0 and 1; locally, however, this is simplifying the matter too much.

# Comments on the Analysis of Free Vibration of Rotationally Symmetric Shells

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IN a recent paper,[1] Stodola's iterative method of finding eigenvalues has been proposed for the analysis of free vibration of shells of revolution. This method is being offered as an alternate to the multisegment free vibration method employed earlier.[2] The writer would like to make some comments on the merits of these two methods.

First of all, the multisegment free vibration method employed in Ref. 2 is not an "iterative" method in the same sense as Stodola's method. According to the multisegment method, the frequency equation of an arbitrary shell of revolution is obtained in the form of a determinant of a $(4 \times 4)$ matrix whose elements, for a given frequency, are determined by means of direct numerical integration of the differential equations. The roots of the frequency equation are found in the same way as the roots of any transcendental algebraic equation. In practice, the finding of the roots is a relatively simple matter, and it involves a systematic evaluation of the frequency equation at selected points within a given frequency interval. As soon as a sign change in the determinant is detected, the natural frequency is determined as accurately as desired by means of inverse interpolation. The point is that no "convergence" is involved in the sense of convergence of an assumed solution toward the actual solution, which is the basis of the usual "iterative" methods, such as Stodola's method.

As is well known,[3] Stodola's method (sometimes called the method of Stodola and Vianello) has been of great practical value in vibration and stability problems of beams. The method starts with a rough estimate of the deflection curve of the beam, from which a better estimate is obtained. It can be proved (Ref. 3, p. 201) that this iteration process is convergent for the lowest eigenvalue. Higher eigenvalues can be obtained by subtracting out from the assumed solution all preceding eigenfunctions.

In the opinion of the writer, the only reason for wanting to apply Stodola's method to shells of revolution is that instead of eight homogeneous solutions, which are needed to find one value of the frequency determinant by the multisegment method, only one particular solution per iteration need be calculated. However, whereas in the multisegment method

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